Pattern Formation in Magnetically Confined Plasmas: Why Staircases are Inevitable

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Contents

A Simpler(?!) Problem:

→ Turbulent Pipe Flow

- Essence of confinement:
  - given device, sources; what profile is achieved?
  - $\tau_E = W/P_{in}$
- Related problem: Pipe flow (turbulent)

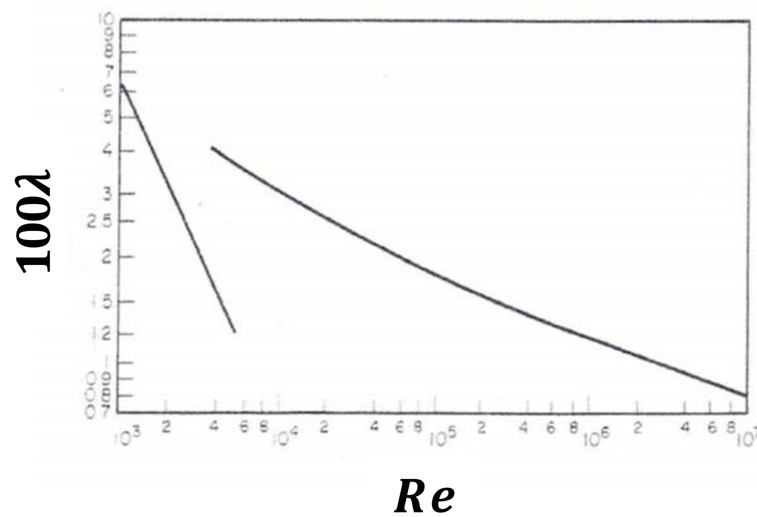
ΔP → pressure drop

l

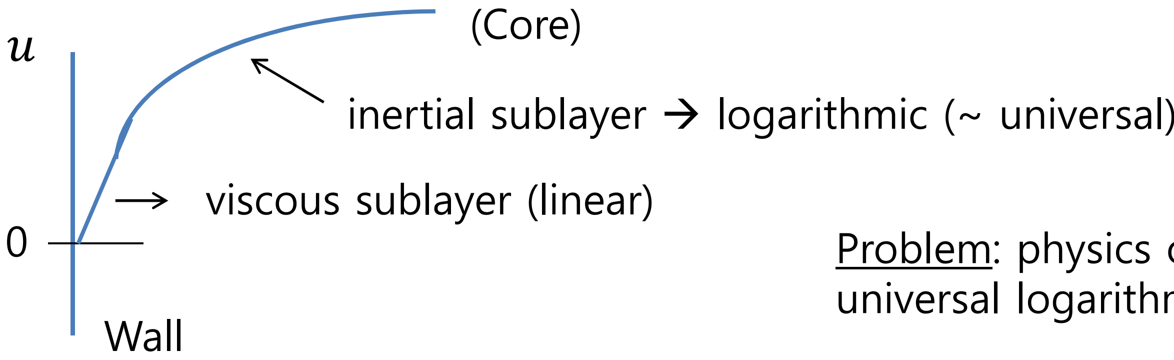
$$\Delta P \pi a^2 = \rho V_*^2 2\pi a l$$

Balance: momentum transport to wall (Reynolds stress) vs ΔP

➔ Flow profile



$$\lambda = \frac{2a\Delta P/l}{1/2\rho u^2}$$



<u>Problem</u>: physics of ~ universal logarithmic profile?

- <u>Prandtl Mixing Length Theory</u> (1932)
  - Wall stress = $\rho V_*^2 = -\rho \nu_T \, \partial u / \partial x$
    ↑ eddy viscosity
  - Absence of characteristic scale →

$$\nu_T \sim V_* x$$

$$u \sim V_* \ln(x/x_0)$$

$x \equiv$ mixing length, distance from wall

Analogy with kinetic theory …

$\nu_T = \nu \rightarrow x_0$, viscous layer → $x_0 = \nu / V_*$

Some key elements:

- Momentum flux driven process
- Turbulent diffusion model of transport eddy viscosity
- Mixing length:

  ~ x → macroscopic, eddys span system

  → ~ flat profile
- Self-similarity in radius
- Cut-off when $\nu_T = \nu$
- Reduce drag by creation of buffer layer i.e. steeper gradient than inertial sublayer (by polymer)

Aside: FYI – Historical Note

- –→ Collective Dynamics of Turbulent Eddy
- – 'Aether' I – First Quasi-Particle Model of Transport?!
- – Kelvin, 1887

XLV. *On the Propagation of Laminar Motion through a turbulently moving Inviscid Liquid.* *By* Sir WILLIAM THOMSON, *LL.D., F.R.S.**

1. IN endeavouring to investigate turbulent motion of water between two fixed planes, for a promised communication to Section A of the British Association at its coming Meeting in Manchester, I have found something seemingly towards a solution (many times tried for within the last twenty years) of the problem to construct, by giving vortex motion to an incompressible inviscid fluid, a medium which shall transmit waves of laminar motion as the luminiferous æther transmits waves of light.

2. Let the fluid be unbounded on all sides, and let u, v, w be the velocity-components, and p the pressure at (x, y, z, t). We have

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad . \quad . \quad . \quad . \quad . \quad (1),$$

* Communicated by the Author, having been read before Section A of the British Association at its recent Meeting in Manchester.

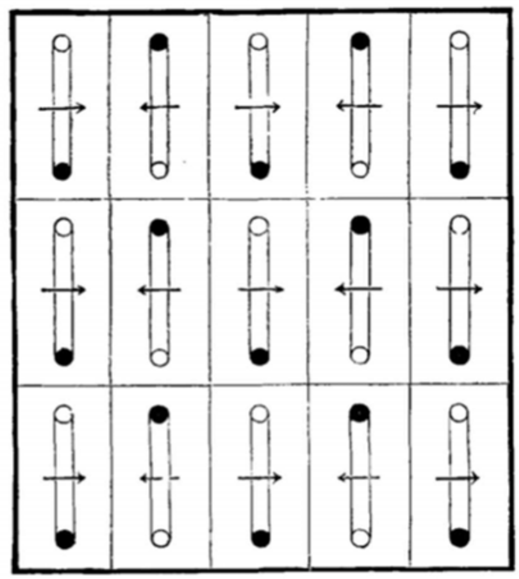
21. Eliminating the first member from this equation, by (34), we find

$$\frac{d^2f}{dt^2} = \frac{2}{3} R^2 \frac{d^2f}{dy^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51).$$

$R^2 \sim \langle \tilde{v}^2 \rangle$

Thus we have the very remarkable result that laminar disturbance is propagated according to the well-known mode of waves of distortion in a homogeneous elastic solid; and that the velocity of propagation is $\frac{\sqrt{2}}{3}$R, or about ·47 of the average velocity of the turbulent motion of the fluid.

Fig. 1.



→ time delay between Reynolds stress and wave shear introduced

→ converts diffusion equation to wave equation

→ describes wave in ensemble of vortex quasi-particles

– c.f. “Worlds of Flow”, O. Darrigol

II) The System:
What is a Tokamak?

How does confinement work?

N.B. No programmatic advertising intended…

Magnetically confined plasma

- Nuclear fusion: option for generating large amounts of carbon-free energy
- Challenge: ignition -- reaction release more energy than the input energy

  Lawson criterion:

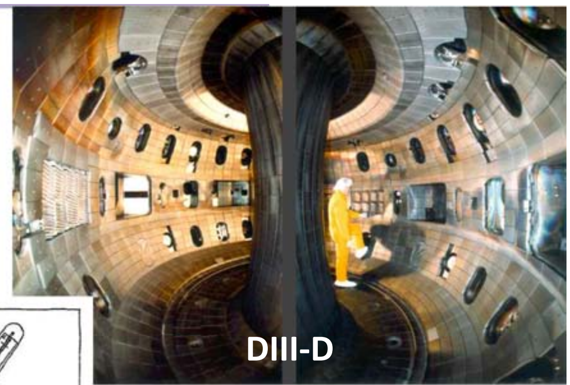
  $$n_i \tau_E T_i > 3\times 10^{21} \mathrm{m}^{-3} \mathrm{s\,keV}$$

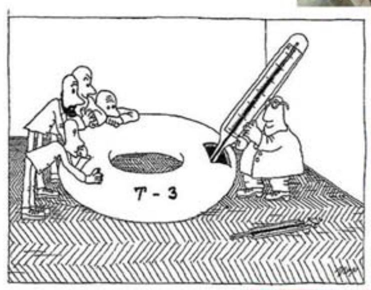
  → confinement

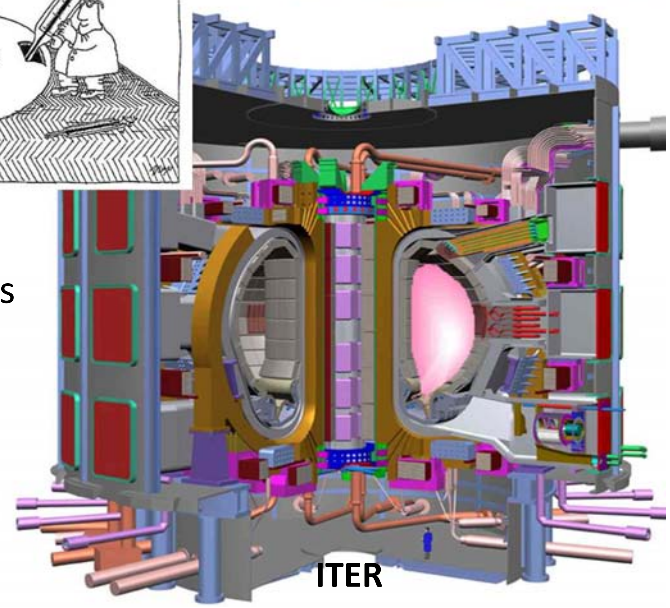
  → turbulent transport
- Turbulence: instabilities and collective oscillations

  → lowest frequency modes dominate the transport

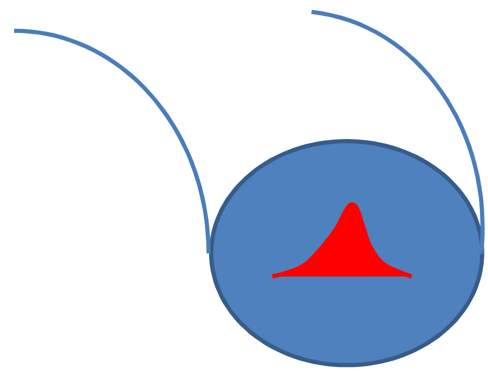
  → drift wave





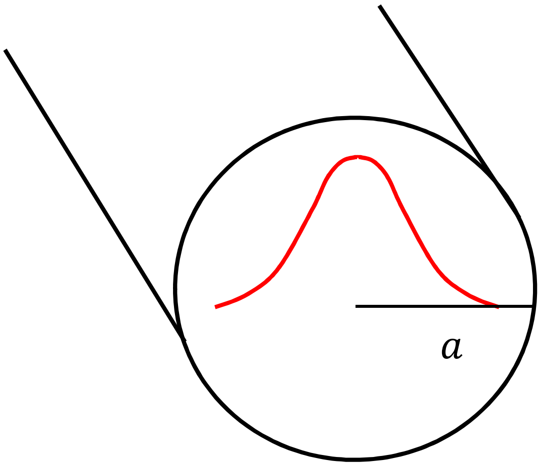


Primer on Turbulence in Tokamaks I



- Strongly magnetized
  - Quasi 2D cells
  - Localized by $\vec{k} \cdot \vec{B} = 0$ (resonance)
- $\vec{V}_\perp = +\frac{c}{B}\,\vec{E} \times \hat{z}$
- ∇T_e, ∇T_i, ∇n driven
- Akin to thermal Rossby wave, with: g → magnetic curvature
- Resembles wave turbulence, not high Re Navier-Stokes turbulence
- Re ill defined, "Re" ≤ 100
- , $K \sim \tilde{V}\tau_c/\Delta \sim 1$ → *Kubo* # ≈ 1
- Broad dynamic range

Primer on Turbulence in Tokamaks II



2 scales:

$\rho \equiv$ gyro-radius

$a \equiv$ cross-section

$\rho_* \equiv \rho/a$ ➔ key ratio

- Characteristic scale ~ few ρ_i → "mixing length"
- Characteristic velocity $v_d \sim \rho_* c_s$

- Transport scaling: $D \sim \rho v_d \sim \rho_* D_B \sim D_{GB}$, $D_B \sim \rho_s c_s$
- i.e. Bigger is better! ➔ sets profile scale via heat balance (Why ITER is huge…)
- Reality: $D \sim \rho_*^\alpha D_B$, $\alpha < 1$ ➔ why??
- 2 Scales, $\rho_* \ll 1$ ➔ key contrast to pipe flow

Drift wave model – Fundamental prototype

- Hasegawa-Wakatani : simplest model incorporating instability

$$V = \frac{c}{B}\hat{z}\times\nabla\phi + V_{pol}$$

$$J_\perp = n|e|V^i_{pol} \qquad \eta J_\parallel = -\nabla_\parallel\phi + \nabla_\parallel p_e$$

$$\nabla_\perp\cdot J_\perp + \nabla_\parallel J_\parallel = 0 \quad \rightarrow \text{vorticity:} \quad \rho_s^2\frac{d}{dt}\nabla^2\phi = -D_\parallel\nabla_\parallel^2(\phi - n) + \nu\nabla^2\nabla^2\phi$$

$$\frac{dn_e}{dt} + \frac{\nabla_\parallel J_\parallel}{-n_0|e|} = 0 \quad \rightarrow \text{density:} \quad \frac{d}{dt}n = -D_\parallel\nabla_\parallel^2(\phi - n) + D_0\nabla^2 n$$

→ PV conservation in inviscid theory $\quad \frac{d}{dt}\left(n - \nabla^2\phi\right) = 0$

→ PV flux = particle flux + vorticity flux

→ zonal flow being a counterpart of particle flux

QL: $\frac{\partial}{\partial t}\langle n\rangle = -\frac{\partial}{\partial r}\langle\tilde{v}_r\tilde{n}\rangle$

→? $\frac{\partial}{\partial t}\langle\nabla^2\phi\rangle = -\frac{\partial}{\partial r}\langle\tilde{v}_r\nabla^2\tilde{\phi}\rangle = -\frac{\partial^2}{\partial r^2}\langle\tilde{v}_r\tilde{v}_\theta\rangle$

- Hasegawa-Mima ($D_\parallel k_\parallel^2/\omega >> 1 \;\rightarrow\; n \sim \phi$)

$$\frac{d}{dt}\left(\phi - \rho_s^2\nabla^2\phi\right) + \upsilon_*\partial_y\phi = 0$$

III) Patterns in Tokamak Turbulence

→ Non-locality and Avalanches
→ Zonal Flows

→ "Truth is never pure and rarely simple" (Oscar Wilde)

Transport: Local or Non-local?

- 40 years of fusion plasma modeling
  - local, diffusive transport

$$Q = -n\chi(r)\nabla T, \quad \chi \leftrightarrow D_{GB}$$

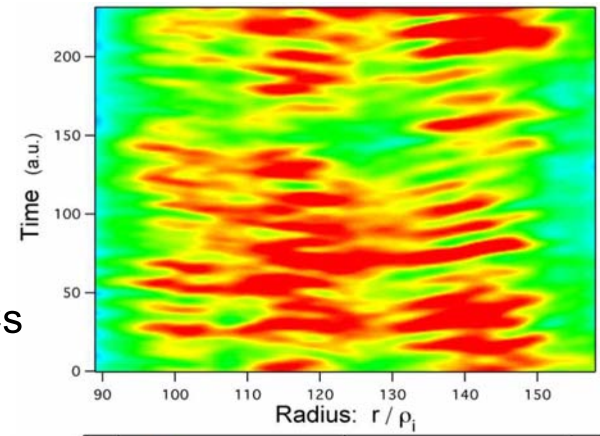
- 1995 → increasing evidence for:
  - transport by avalanches, as in sand pile/SOCs
  - turbulence propagation and invasion fronts
  - "non-locality of transport"

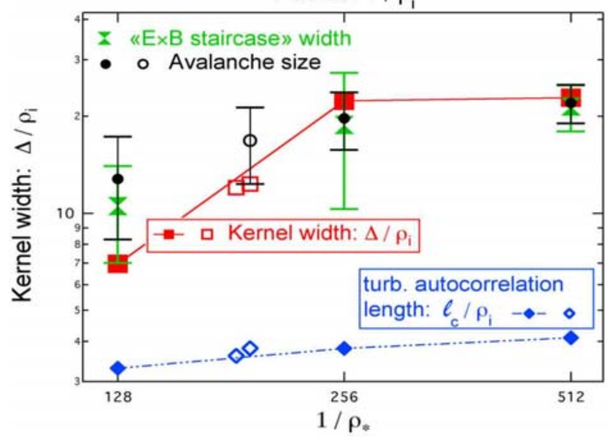
$$Q = -\int \kappa(r,r')\nabla T(r')dr'$$

$$\kappa(r,r') \sim S_0 / \left[(r-r')^2 + \Delta^2\right]$$

- Physics:
  - Levy flights, SOC, turbulence fronts…
- Fusion:
  - gyro-Bohm breaking (ITER: significant ρ_* extension)

→ *fundamentals of turbulent transport modeling??*





Guilhem Dif-Pradalier et al. PRL 2009

Observe:

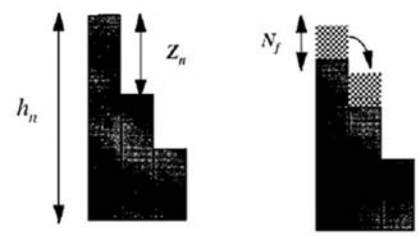
- Cells “pinned” by magnetic geometry → resonances
- Remarkable

  Similarity:

TABLE I. Analogies between the sandpile transport model and a turbulent transport model.

Turbulent transport in toroidal plasmas	Sandpile model
Localized fluctuation (eddy)	Grid site (cell)
Local turbulence mechanism:	*Automata rules*:
Critical gradient for local instability	Critical sandpile slope (Z_{crit})
Local eddy-induced transport	Number of grains moved if unstable (N_f)
Total energy/particle content	Total number of grains (total mass)
Heating noise/background fluctuations	Random rain of grains
Energy/particle flux	Sand flux
Mean temperature/density profiles	Average slope of sandpile
Transport event	Avalanche
Sheared electric field	Sheared flow (sheared wind)

Automaton toppling
↔ Cell/eddy overturning

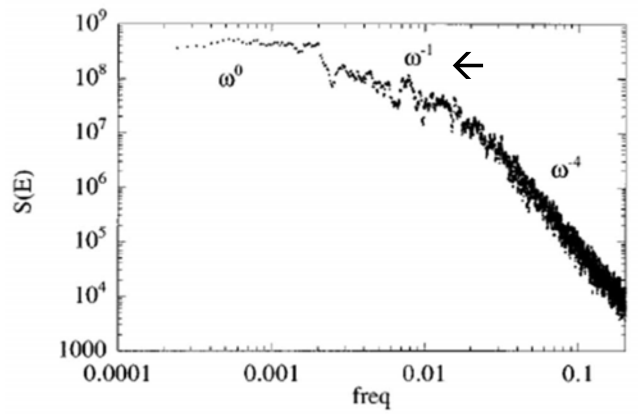


and can cooperate!

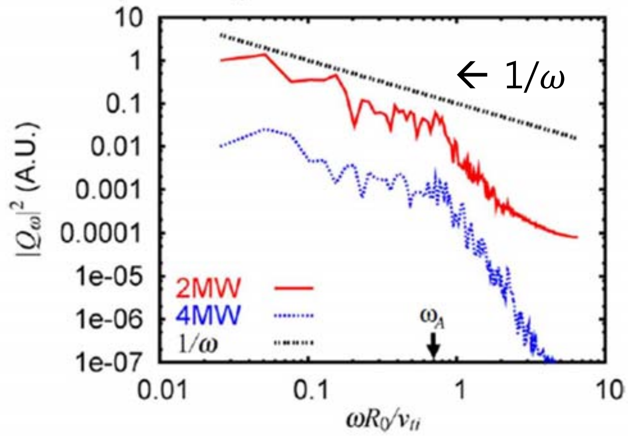
→ Avalanches happen!

FIG. 1. A cartoon representation of the simple cellular automata rules used to model the sandpile.

- ‘Avalanches’ form! – flux drive + geometrical ‘pinning’



Newman PoP96 (sandpile)
(Autopower frequency spectrum of ‘flip’)



GK simulation also exhibits avalanching
(Heat Flux Spectrum) (Idomura NF09)

- Avalanching is a likely cause of ‘gyro-Bohm breaking’ → Intermittent Bursts

➔ localized cells self-organize to form transient, extended transport events

- Akin domino toppling:
- Pattern competition with shear flows!

Toppling front can penetrate beyond region of local stability

Shear Flows !?

No shear

shear

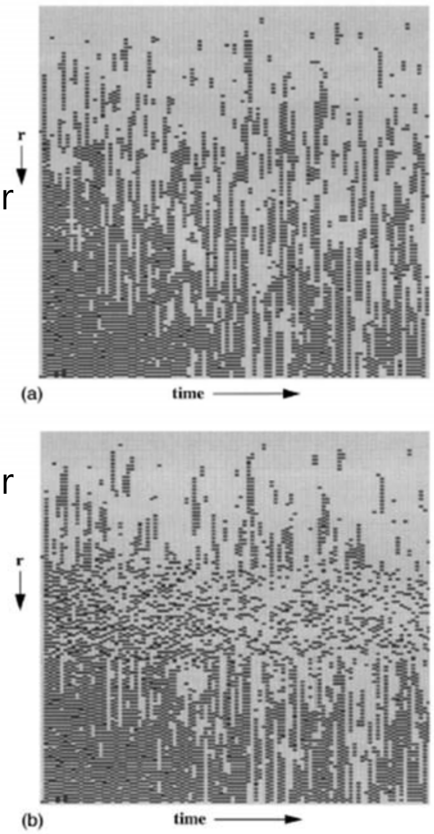


FIG. 11. Time evolution of the overturning sites (like Fig. 4). The avalanches do not appear continous in time because only every 50th time step is shown. (a) The shear-free case shows avalanches of all lengths over the entire radius. (b) The case with sheared flow shows the coherent avalanches being decorrelated in the shear zone in the middle of the pile.

How is transport suppressed?

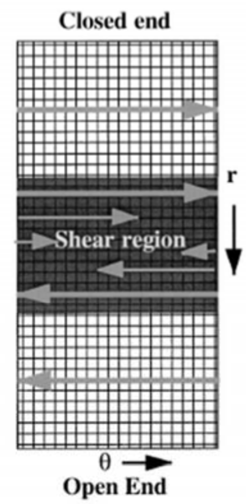
➔ shear decorrelation!

Back to sandpile model:

2D pile +

sheared flow of

grains



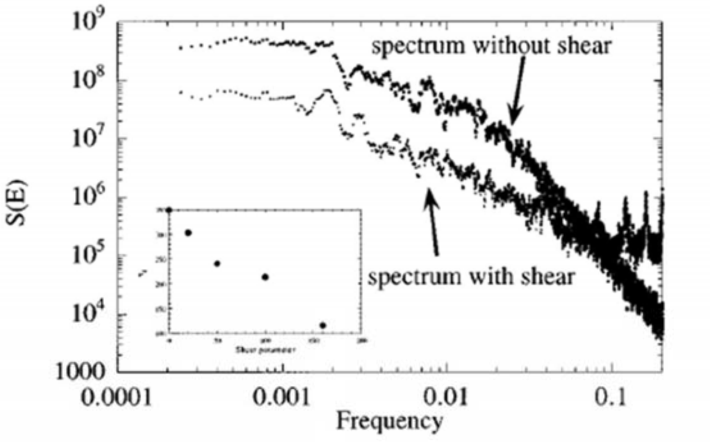
Shearing flow
decorrelates
Toppling sequence

FIG. 10. A cartoon of the sandpile with a shear flow zone. The whole pile is flowing to the right at the top and to the left at the bottom connected by a variable sized region of sheared flow.

Avalanche coherence destroyed by shear flow

- Implications:

Spectrum of Avalanches



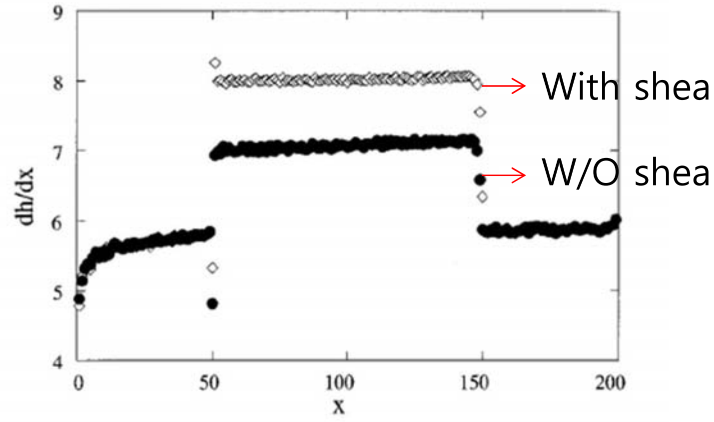
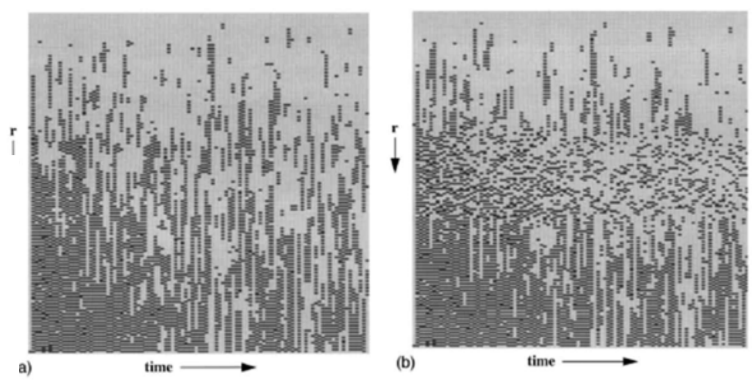


FIG. 14. The slopes of a sandpile with a shear region in the middle, including all the shear effects (diamonds) and just the transport decorrelation and the linear effect (circles).



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N.B.

- Profile steepens for <u>unchanged</u> toppling rules
- <u>Distribution</u> of avalanches fundamental

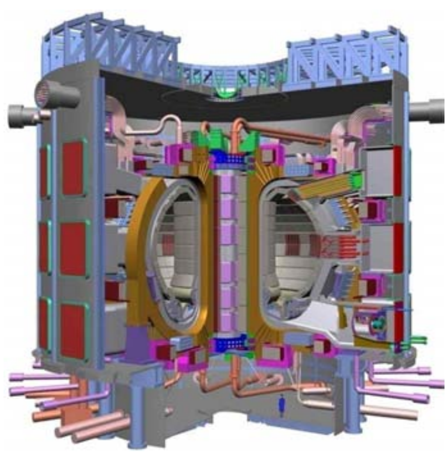
Shear Flows ‘Natural’ to Tokamaks

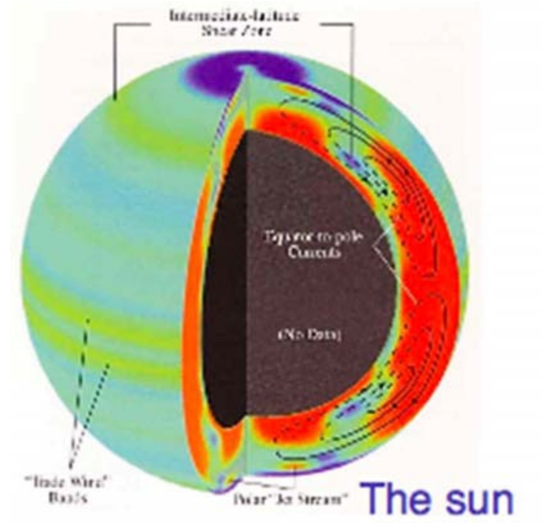
- Zonal Flows Ubiquitous for:

  ~ 2D fluids / plasmas $R_0 < 1$

  Rotation $\vec{\Omega}$, Magnetization $\vec{B}_0$, Stratification

  Ex: MFE devices, giant planets, stars…

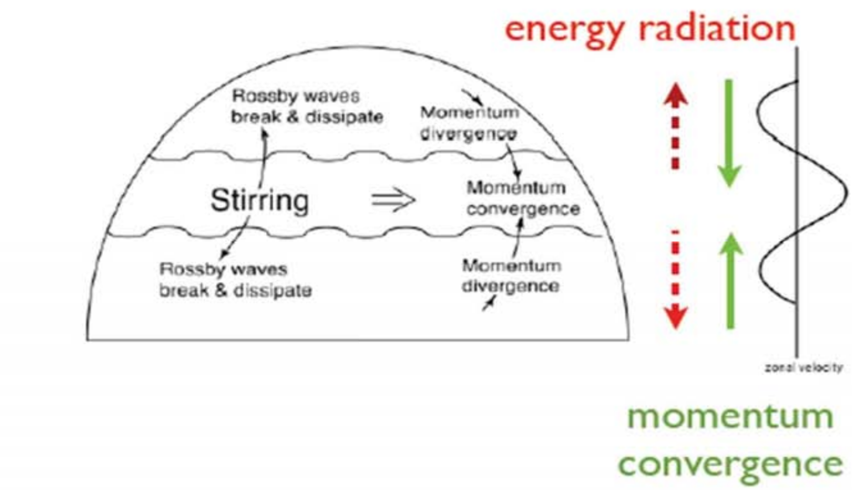




Heuristics of Zonal Flows a): How Form?

Simple Example: Zonally Averaged Mid-Latitude Circulation

- classic GFD example: Rossby waves + Zonal flow (c.f. Vallis '07, Held '01)
- Key Physics:



Rossby Wave:

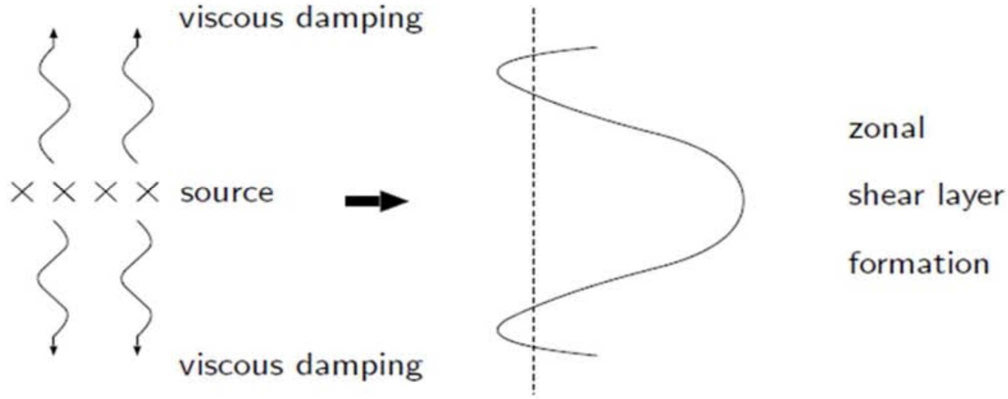
$$\omega_k = -\frac{\beta k_x}{k_\perp^2}$$

$$v_{gy} = 2\beta \frac{k_x k_y}{(k_\perp^2)^2}, \ \langle \tilde{v}_y \tilde{v}_x \rangle = \sum_{\vec{k}} -k_x k_y |\hat{\phi}_{\vec{k}}|^2$$

$\therefore v_{gy} v_{phy} < 0$ → Backward wave!

➔ Momentum convergence at stirring location

- ..."the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into this region." (I. Held, '01)
- Outgoing waves ⇒ incoming wave momentum flux

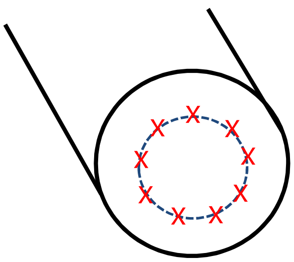


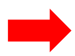
- Local Flow Direction (northern hemisphere):
  - eastward in source region
  - westward in sink region
  - set by $\beta > 0$
  - Some similarity to spinodal decomposition phenomena
    → Both 'negative diffusion' phenomena

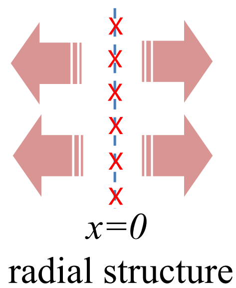
Wave-Flows in Plasmas

MFE perspective on Wave Transport in DW Turbulence

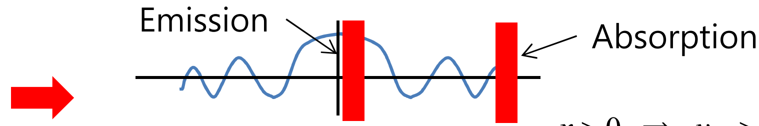
- localized source/instability drive intrinsic to drift wave structure







  – couple to damping ↔ outgoing wave

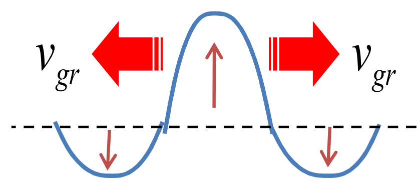


  – $v_{gr} = -2\rho_s^2 \dfrac{k_\theta k_r v_*}{(1+k_\perp^2\rho_s^2)^2}$ $\quad x > 0 \;\Rightarrow\; v_{gr} > 0$

  $v_* < 0 \;\Rightarrow\; k_r k_\theta > 0$

  $\langle v_{rE} v_{\theta E} \rangle = -\dfrac{c^2}{B^2} |\phi_{\vec{k}}|^2 \, k_r k_\theta < 0$

- outgoing wave energy flux → incoming wave momentum flux → counter flow spin-up!



- zonal flow layers form at excitation regions

Zonal Flows I

- What is a Zonal Flow?
  - *n = 0* potential mode; *m = 0* (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric *ExB* shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport (*n = 0*)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

PV conservation

- **PV conservation dq/dt=0**

GFD: Quasi-geostrophic system	Plasma: Hasegawa-Wakatani system
$q = \nabla^2 \psi + \beta y$ relative vorticity ($\nabla^2 \psi$), planetary vorticity (βy)	$q = n - \nabla^2 \phi$ density (guiding center) (n), ion vorticity (polarization) ($\nabla^2 \phi$)
Physics: $\Delta y \to \Delta\left(\nabla^2 \psi\right)$	Physics: $\Delta r \to \Delta n \to \Delta\left(\nabla^2 \phi\right)$ ZF!

- Charney-Haswgawa-Mima equation

$$n = n_0 + \tilde{n}$$
$$\tilde{n} \sim \frac{e\tilde{\phi}}{T}$$

H-W → H-M: $$\frac{1}{\omega_{ci}}\frac{\partial}{\partial t}\left(\nabla^2\phi - \rho_s^{-2}\phi\right) - \frac{1}{L_n}\frac{\partial}{\partial y}\phi + \frac{\rho_s}{L_n}J(\phi, \nabla^2\phi) = 0$$

Q-G: $$\frac{\partial}{\partial t}\left(\nabla^2\psi - L_d^{-2}\psi\right) + \beta\frac{\partial}{\partial x}\psi + J(\psi, \nabla^2\psi) = 0$$

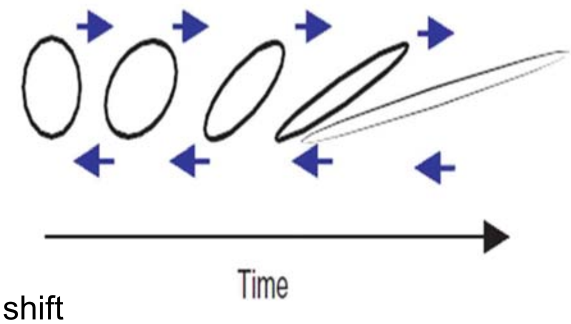
Zonal Flows II

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

- G.C. ambipolarity breaking → polarization charge flux → Reynolds force
  - Polarization charge ➡ $-\rho^2\nabla^2\phi = n_{i,GC}(\phi) - n_e(\phi)$
    polarization length scale (ρ^2) ; *ion GC* ($n_{i,GC}$) ; *electron density* (n_e)
  - so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ $\rho^2\langle\tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\rangle \neq 0$ ⬌ 'PV transport'
    polarization flux → What sets cross-phase?
  - If 1 direction of symmetry (or near symmetry):
    $-\rho^2\langle\tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\rangle = -\partial_r\langle\tilde{v}_{rE}\tilde{v}_{\perp E}\rangle$ (Taylor, 1915)
    $-\partial_r\langle\tilde{v}_{rE}\tilde{v}_{\perp E}\rangle$ ➡ Reynolds force ➡ Flow

Zonal Flows Shear Eddys I

- Coherent shearing: (Kelvin, G.I. Taylor, Dupree'66, BDT'90)
  - radial scattering + $\langle V_E \rangle' \rightarrow$ hybrid decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1/\tau_c$
  - shaping, flux compression: Hahm, Burrell '94



- Other shearing effects (linear):
  - spatial resonance dispersion: $\omega - k_\parallel v_\parallel \Rightarrow \omega - k_\parallel v_\parallel - k_\theta \langle V_E \rangle' (r - r_0)$

    Response shift and dispersion
  - differential response rotation → especially for kinetic curvature effects

→ N.B. Caveat: Modes can adjust to weaken effect of external shear

(Carreras, et. al. '92; Scott '92)

Shearing II

- Zonal Shears: Wave kinetics (Zakharov et. al.; P.D. et. al. '98, et. seq.)

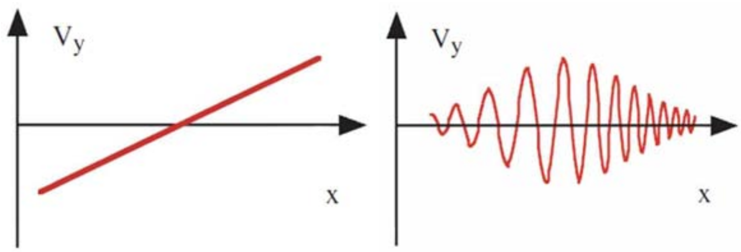
  Coherent interaction approach (L. Chen et. al.)

- $dk_r / dt = -\partial(\omega + k_\theta V_E)/\partial r\,;\ V_E = \langle V_E \rangle + \widetilde{V}_E$

  Mean shearing $: k_r = k_r^{(0)} - k_\theta V_E' \tau$

  Zonal Random shearing $: \langle \delta k_r^2 \rangle = D_k \tau$

  $D_k = \sum_q k_\theta^2 \left| \widetilde{V}'_{E,q} \right|^2 \tau_{k,q}$



  - Wave ray chaos (not shear RPA) underlies $D_k \rightarrow$ induced diffusion
  - Induces wave packet dispersion
  - Applicable to ZFs and GAMs

- Mean Field Wave Kinetics

$$\frac{\partial N}{\partial t} + (\vec{V}_{gr} + \vec{V}) \cdot \nabla N - \frac{\partial}{\partial r}(\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial \vec{k}} = \gamma_{\vec{k}} N - C\{N\}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N \rangle = \gamma_{\vec{k}} \langle N \rangle - \langle C\{N\} \rangle}$$

Zonal shearing

Shearing III

- Energetics: Books Balance for Reynolds Stress-Driven Flows!
- Fluctuation Energy Evolution – Z.F. shearing

$$\int d\vec{k}\,\omega\left(\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle\right) \Rightarrow \frac{\partial}{\partial t}\langle \varepsilon\rangle = -\int d\vec{k}\, V_{gr}(\vec{k}) D_{\vec{k}} \frac{\partial}{\partial k_r}\langle N\rangle \qquad V_{gr} = \frac{-2k_r k_\theta V_* \rho_s^2}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

Point: For $d\langle\Omega\rangle / dk_r < 0$, Z.F. shearing damps wave energy

- Fate of the Energy: Reynolds work on Zonal Flow

Modulational Instability

$$\partial_t \delta V_\theta + \partial\left(\delta\left\langle \tilde{V}_r \tilde{V}_\theta\right\rangle\right)/\partial r = -\gamma\delta V_\theta$$

$$\delta\left\langle \tilde{V}_r \tilde{V}_\theta\right\rangle \sim \frac{k_r k_\theta \delta\Omega}{(1+k_\perp^2\rho_s^2)^2}$$

N.B.: Wave decorrelation essential: Equivalent to PV transport (c.f. Gurcan et. al. 2010)

- Bottom Line:
  - Z.F. growth due to shearing of waves
  - "Reynolds work" and "flow shearing" as relabeling → books balance
  - Z.F. damping emerges as critical; MNR '97

Non-perturbative approaches

- PV mixing in space is essential in ZF generation.

Taylor identity: $\left\langle \tilde{\upsilon}_y \nabla^2 \tilde{\phi} \right\rangle = -\partial_y \left\langle \tilde{\upsilon}_y \tilde{\upsilon}_x \right\rangle$

vorticity flux   Reynolds force

Key:
How represent inhomogeneous PV mixing

→ **General structure** of PV flux?
→relaxation principles!

most treatment of ZF:
-- perturbation theory
-- modulational instability
(test shear + gas of waves)
~ linear theory

-> physics of evolved PV mixing?
-> something more general?

non-perturb model 1: use selective decay principle

What form must the PV flux have so as to dissipate enstrophy while conserving energy?

non-perturb model 2: use joint reflection symmetry

What form must the PV flux have so as to satisfy the joint reflection symmetry principle for PV transport/mixing?

General principle: selective decay

- 2D turbulence conservation of energy and potential enstrophy

→ dual cascade

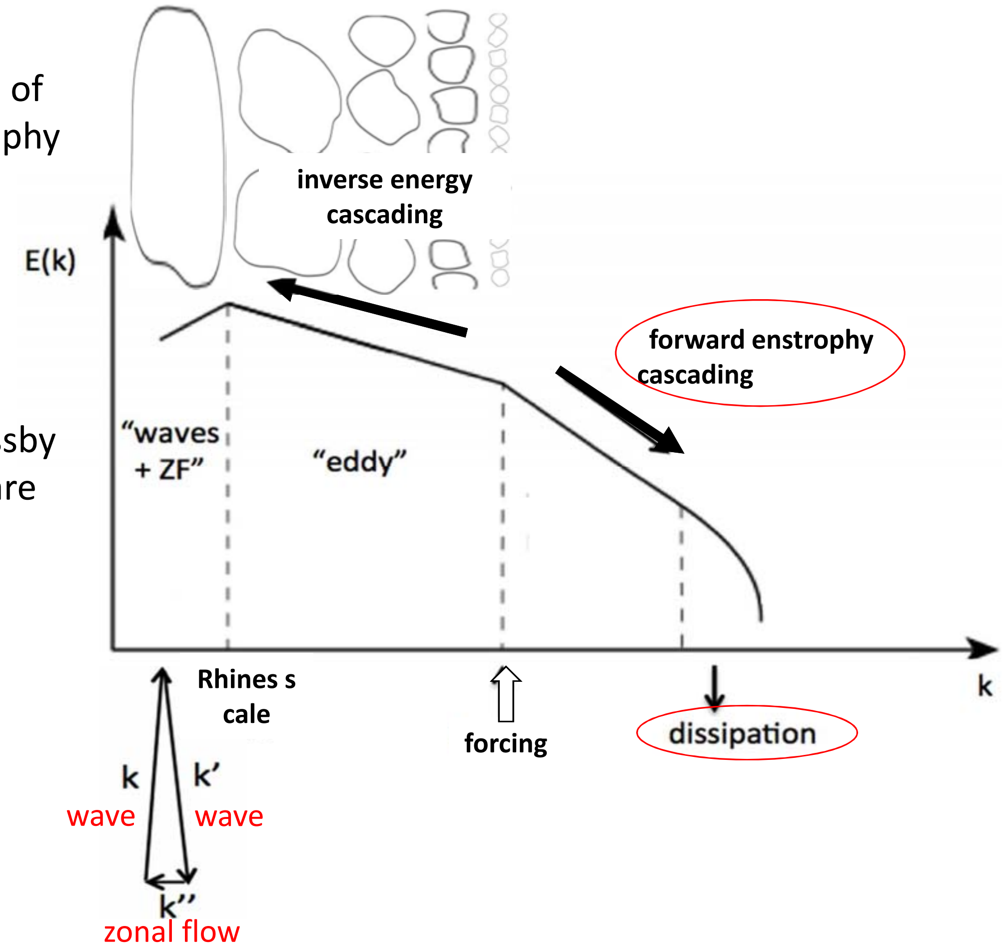
→ Minimum enstrophy state

- eddy turnover rate and Rossby wave frequency mismatch are comparable

$$\frac{\partial \omega}{\partial t} + \bar{u} \cdot \nabla \omega + \beta v = 0$$

$$\frac{U}{LT} \quad \frac{U^2}{L^2} \quad \beta U$$

→ Rhines scale $L_R \sim \sqrt{\frac{U}{\beta}}$



Using selective decay for flux

	minimum enstrophy relaxation (Bretherton & Haidvogel 1976)	analogy ⟷	Taylor relaxation (J.B. Taylor, 1974)
turbulence	2D hydro		3D MHD
conserved quantity (constraint)	total kinetic energy		global magnetic helicity
dissipated quantity (minimized)	fluctuation potential enstrophy		magnetic energy
final state	minimum enstrophy state flow structure emergent		Taylor state force free B field configuration
structural approach	$\frac{\partial}{\partial t}\Omega < 0 \Rightarrow \Gamma_E \Rightarrow \Gamma_q$		$\frac{\partial}{\partial t}E_M < 0 \Rightarrow \Gamma_H$

dual cascade (conserved quantity and dissipated quantity rows)

- flux? what can be said about dynamics?

→ structural approach (this work): *What form must the PV flux have so as to dissipate enstrophy while conserving energy?*

General principle based on general physical ideas → useful for dynamical model

PV flux

→ PV conservation

mean field PV: $$\frac{\partial\langle q\rangle}{\partial t}+\partial_y\underbrace{\langle \upsilon_y q\rangle}_{\Gamma_q}=\nu_0\partial_y^2\langle q\rangle$$

Γ_q : mean field PV flux

Key Point: what form does PV flux have s/t dissipate enstrophy, conserve energy

selective decay

→ energy conserved $$E=\int\frac{\left(\partial_y\langle\phi\rangle\right)^2}{2}$$

$$\frac{\partial E}{\partial t}=\int\langle\phi\rangle\partial_y\Gamma_q=-\int\partial_y\langle\phi\rangle\Gamma_q \qquad \Rightarrow\Gamma_q=\frac{\partial_y\Gamma_E}{\partial_y\langle\phi\rangle}$$

→ enstrophy minimized $$\Omega=\int\frac{\langle q\rangle^2}{2}$$

$$\frac{\partial\Omega}{\partial t}=-\int\langle q\rangle\partial_y\Gamma_q=-\int\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right)\Gamma_E$$

$$\frac{\partial\Omega}{\partial t}<0\Rightarrow\Gamma_E=\mu\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right) \qquad \Rightarrow\Gamma_q=\frac{1}{\partial_y\langle\phi\rangle}\partial_y\left[\mu\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right)\right]$$

general form of PV flux

μ: parameter TBD

$\partial_y\langle\phi\rangle \to \langle\upsilon_x\rangle$

Structure of PV flux

$$\Gamma_q = \frac{1}{\langle \upsilon_x \rangle}\partial_y\left[\mu\,\partial_y\left(\frac{\partial_y\langle q\rangle}{\langle \upsilon_x\rangle}\right)\right] = \frac{1}{\langle \upsilon_x \rangle}\partial_y\left[\mu\left(\frac{\langle q\rangle\partial_y\langle q\rangle}{\langle \upsilon_x\rangle^2}+\frac{\partial_y^2\langle q\rangle}{\langle \upsilon_x\rangle}\right)\right]$$

diffusion parameter calculated by pert urbation theory, numerics…

diffusion and hyper diffusion of PV

<--> usual story : Fick's diffusion

relaxed state:

Homogenization of $\frac{\partial_y\langle q\rangle}{\langle \upsilon_x\rangle}$ → allows staircase

characteristic scale $\ell_c \equiv \sqrt{\left|\frac{\langle \upsilon_x\rangle}{\partial_y\langle q\rangle}\right|}$

$\ell > \ell_c$: zonal flow growth

$\ell < \ell_c$: zonal flow damping (hyper viscosity-dominated)

Rhines scale $L_R \sim \sqrt{\frac{U}{\beta}}$

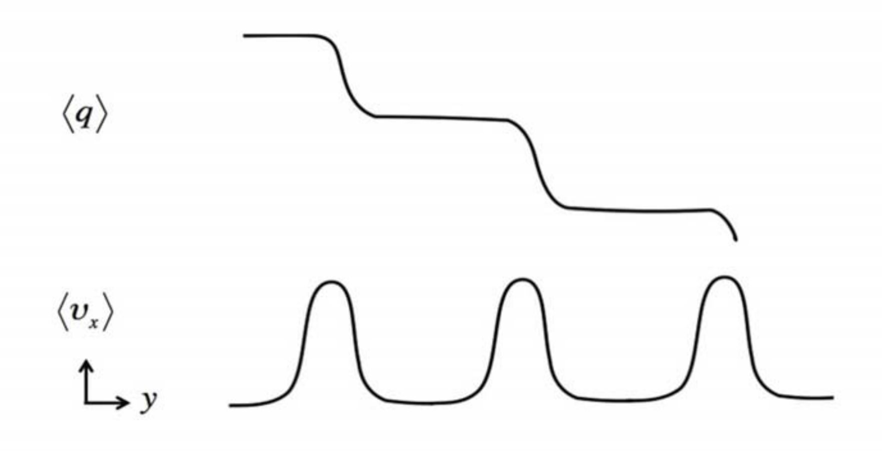
$\ell > L_R$: wave-dominated

$\ell < L_R$: eddy-dominated

PV staircase

relaxed state: homogenization of $\frac{\partial_y \langle q \rangle}{\langle v_x \rangle}$ → PV gradient large where zonal flow large

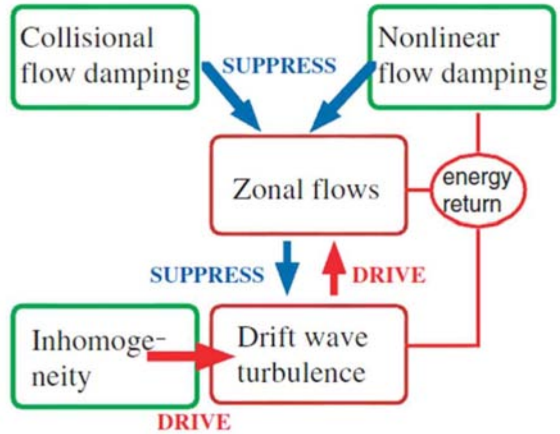
→ Zonal flows track the PV gradient → PV staircase



- Highly structured profile of the staircase is reconciled with the homoge nization or mixing process required to produce it.
- Staircase may arise naturally as a consequence of minimum enstrophy relaxation.

Feedback Loops I

- Closing the loop of shearing and Reynolds work



- Spectral 'Predator-Prey' equations

Prey → Drift waves, <*N*>

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k \langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

Feedback Loops II

- Recovering the 'dual cascade':
  - Prey → <N> ~ <Ω> ⇒ induced diffusion to high k_r { ⇒ Analogous → forward potential enstrophy cascade; PV transport
  - Predator → $|\phi_q|^2 \sim \langle V_{E,\theta}^2 \rangle$ { ⇒ growth of *n=0, m=0* Z.F. by turbulent Reynolds work; ⇒ Analogous → inverse energy cascade

- Mean Field Predator-Prey Model

  (P.D. et. al. '94, DI²H '05)

$$\frac{\partial}{\partial t} N = \gamma N - \alpha V^2 N - \Delta\omega N^2$$

$$\frac{\partial}{\partial t} V^2 = \alpha N V^2 - \gamma_d V^2 - \gamma_{NL}(V^2) V^2$$

System Status

State	No flow	Flow ($\alpha_2 = 0$)	Flow ($\alpha_2 \neq 0$)
N (drift wave turbulence level)	$\frac{\gamma}{\Delta\omega}$	$\frac{\gamma_d}{\alpha}$	$\frac{\gamma_d + \alpha_2 \gamma \alpha^{-1}}{\alpha + \Delta\omega \alpha_2 \alpha^{-1}}$
V^2 (mean square flow)	0	$\frac{\gamma}{\alpha} - \frac{\Delta\omega\gamma_d}{\alpha^2}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
Drive/excitation mechanism	Linear growth	Linear growth	Linear growth Nonlinear damping of flow
Regulation/inhibition mechanism	Self-interaction of turbulence	Random shearing, self-interaction	Random shearing, self-interaction
Branching ratio $\frac{V^2}{N}$	0	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d + \alpha_2\gamma\alpha^{-1}}$
Threshold (without noise)	$\gamma > 0$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$

IV) The Central Question: Secondary Pattern Selection ?!

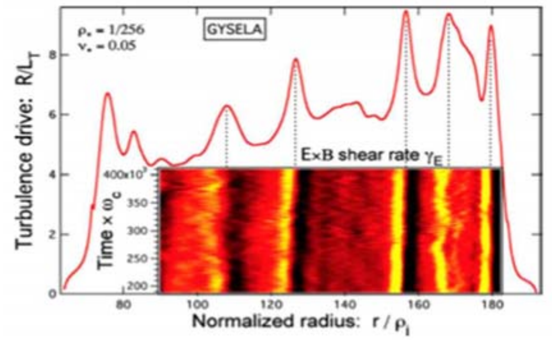
- Two secondary structures suggested
  - Zonal flow → quasi-coherent, regulates transport via shearing
  - Avalanche → stochastic, induces extended transport events
- Both flux driven… by relaxation
- <u>Nature of co-existence??</u>
- Who wins? Does anybody win?

V) Staircases and Traffic Jams

Single Barrier → Lattice of Shear Layers

→ Jam Patterns

Highlights



Observation of ExB staircases

→ Failure of conventional theory

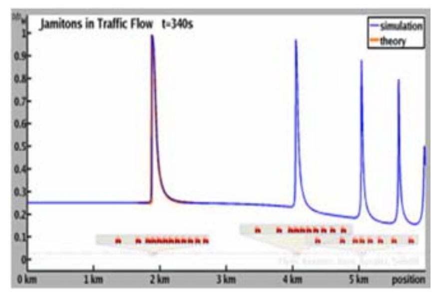
(emergence of particular scale???)

Model extension from Burgers to telegraph

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

$$\Rightarrow \tau \partial_t^2 \delta T + \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

finite response time → like drivers' response time in traffic



Analysis of telegraph eqn. predicts heat flux jam

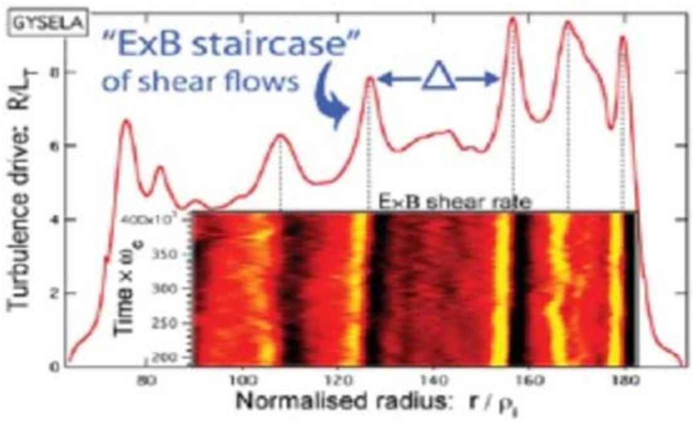
- scale of jam comparable to staircase step

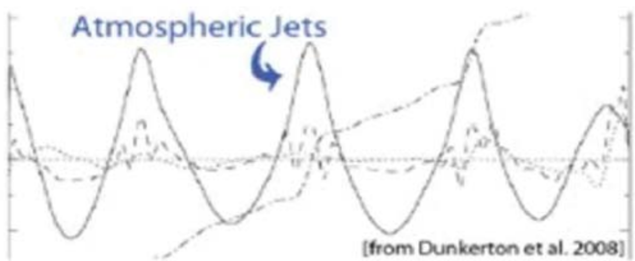
Motivation: ExB staircase formation (1)

- ExB flows often observed to self-organize in magnetized plasmas

  eg. mean sheared flows, zonal flows, ...

- `ExB staircase' is observed to form (G. Dif-Pradalier, P.D. et al. Phys. Rev. E. '10)





- flux driven, full f simulation

- Quasi-regular pattern of shear layers and profile corrugations

- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

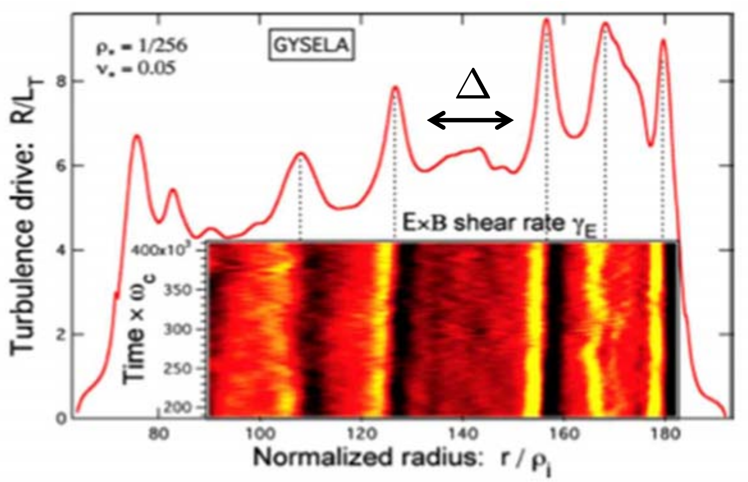
  → ExB staircases

- so-named after the analogy to PV staircases and atmospheric jets

- Step spacing → avalanche outer-scale

ExB Staircase (2)

- Important feature: co-existence of shear flows and avalanches



- Seem mutually exclusive ?!?

  → strong ExB shear prohibits transport

  → avalanches smooth out corrugations

- Can co-exist by separating regions into:

  1. avalanches of the size $\Delta \gg \Delta_c$

  2. localized strong corrugations + jets

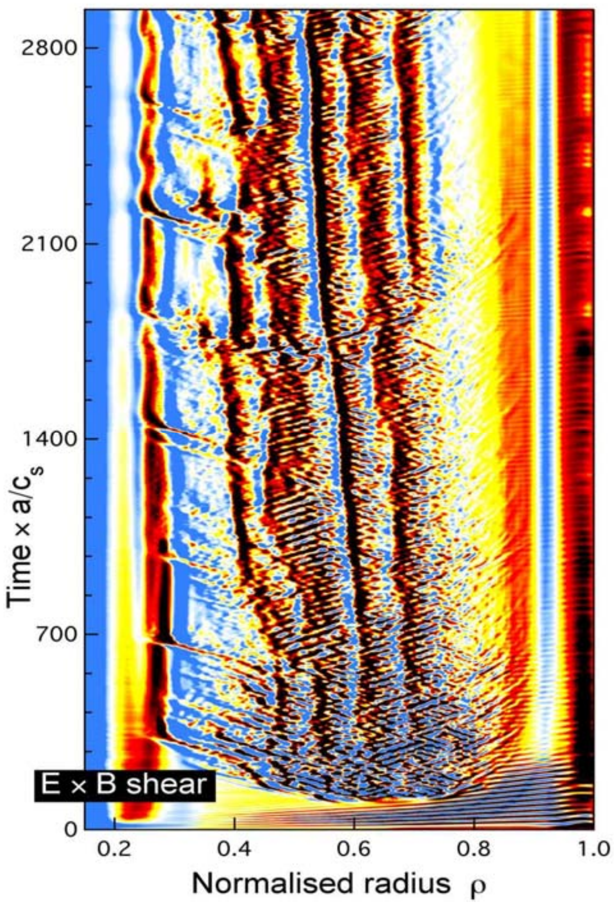
- How understand the formation of ExB staircase???

  - What is process of self-organization linking avalanche scale to ExB step scale?

  i.e. how explain the emergence of the step scale ???

Some Observations:

Staircases build up from the edge

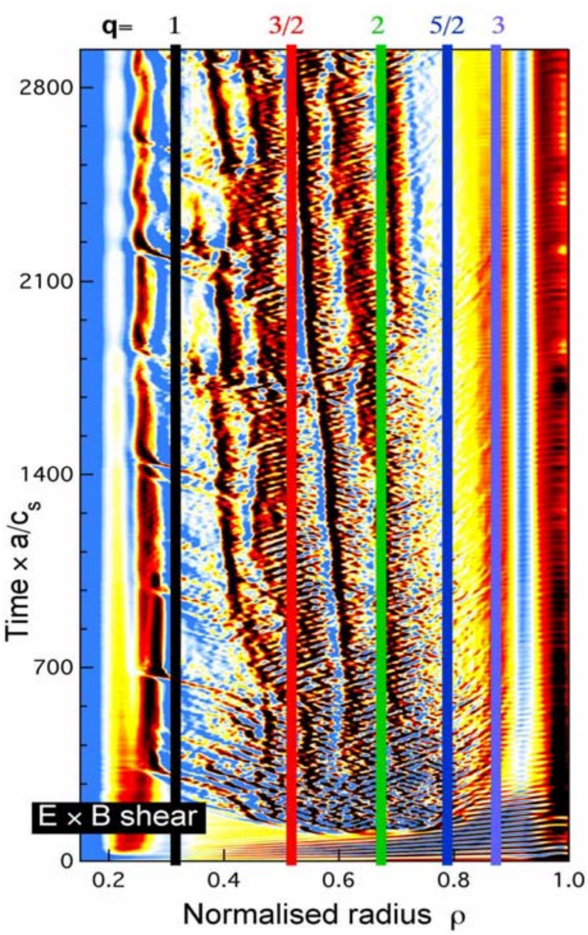


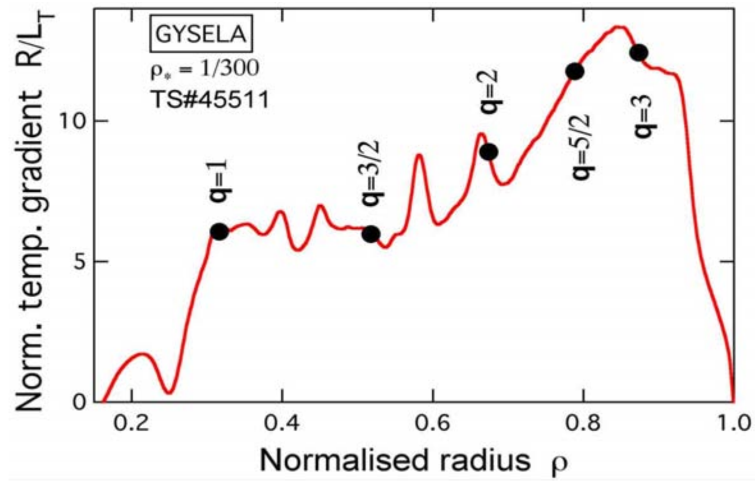
→ staircases may not be related to zonal flow eigenfunctions

→ How describe generation mechanism??

(GYSELA simulation)

Corrugation points and rational surfaces – no relation!



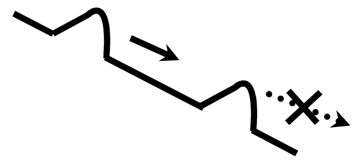


Step location not tied to magnetic geometry structure in a simple way

Towards a model

• How do we understand quasi-regular pattern of ExB staircase, generated from stochastic heat avalanche???

• An idea: jam of heat avalanche



→ accumulation of heat increment
→ stationary corrugated profile

corrugated profile ⟷ ExB staircase

→ corrugation of profile occurs by 'jam' of heat avalanche flux

* → time delay between $Q[\delta T]$ and δT is crucial element

like drivers' response time in traffic

• How do we actually model heat avalanche 'jam'??? → origin in dynamics?

Traffic jam dynamics: ‘jamiton’

- A model for Traffic jam dynamics → Whitham

$$\rho_t + (\rho v)_x = 0$$

$$v_t + v v_x = -\frac{1}{\tau}\left\{v - V(\rho) + \frac{\nu}{\rho}\rho_x\right\}$$

ρ → car density

v → traffic flow velocity

$V(\rho) - \frac{\nu}{\rho}\rho_x$ → an equilibrium traffic flow

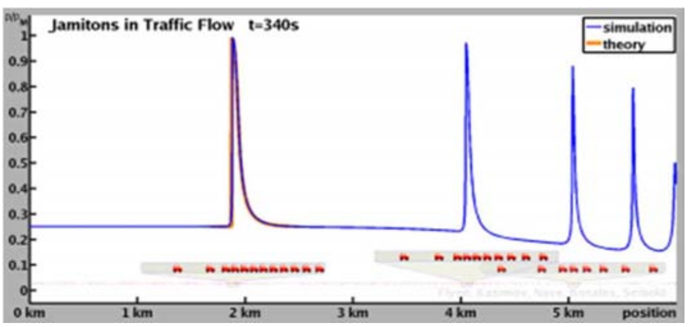
τ → driver’s response time

→ Instability occurs when $\tau > \nu/(\rho_0^2 V_0'^2)$

$D_{eff} = \nu - \tau \rho_0^2 V_0'^2 < 0$ → clustering instability

→ Indicative of jam formation

- Simulation of traffic jam formation



http://math.mit.edu/projects/traffic/

→ Jamitons (Flynn, et.al., ’08)

n.b. I.V.P. → decay study

Heat avalanche dynamics model (`the usual')

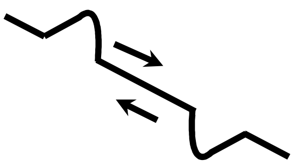
Hwa+Kardar ' 92, P.D. + Hahm ' 95, Carreras, et al. ' 96, ... GK simulation, ... Dif-Pradalier ' 10

- δT :deviation from marginal profile → conserved order parameter

- Heat Balance Eq.: $\partial_t \delta T + \partial_x Q[\delta T] = 0$ → up to source and noise

- Heat Flux $Q[\delta T]$? → utilize symmetry argument, ala' Ginzburg-Landau

  - Usual: → joint reflectional symmetry (Hwa+Kardar'92, Diamond+Hahm '95)



$$\delta T \leftrightarrow -\delta T$$
$$x \leftrightarrow -x$$

$$Q = Q_0(\delta T) = \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

→ hyperdiffusion

lowest order → Burgers equation

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

An extension of the heat avalanche dynamics

- An extension: a finite time of relaxation of Q toward SOC flux state

$$\partial_t Q = -\frac{1}{\tau}\left(Q - Q_0(\delta T)\right) \qquad Q_0[\delta T] = \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

(Guyot-Krumhansl)

→ In principle $\tau(\delta T, Q_0)$ ⟷ large near criticality (∼ critical slowing down)

i.e. enforces time delay between δT and heat flux

N.B.: Contrast quasi-linear theory!

- Dynamics of heat avalanche:

n.b. model for heat evolution

diffusion → Burgers → Telegraph

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \chi_4 \partial_x^4 \delta T - \tau \partial_t^2 \delta T$$

→ Burgers
(P.D. + T.S.H. ' 95)

New: finite response time

→ Telegraph equation

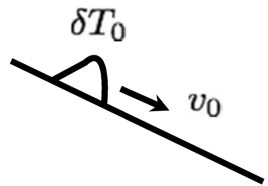
Relaxation time: the idea

- What is 'τ ' physically? → Learn from traffic jam dynamics
- A useful analogy:

heat avalanche dynamics	traffic flow dynamics
temp. deviation from marginal profile	local car density
heat flux	traffic flow
mean SOC flux (ala joint relflection symmetry)	equilibrium, steady traffic flow
➡ heat flux relaxation time	driver's response time

- driver's response can induce traffic jam
- jam in avalanche → profile corrugation → staircase?!?
- Key: instantaneous flux vs. mean flux

Analysis of heat avalanche dynamics via telegraph



- How do heat avalanches jam?

- Consider an initial avalanche, with amplitude δT_0, propagating at the speed $v_0 = \lambda \delta T_0$

→ turbulence model dependent

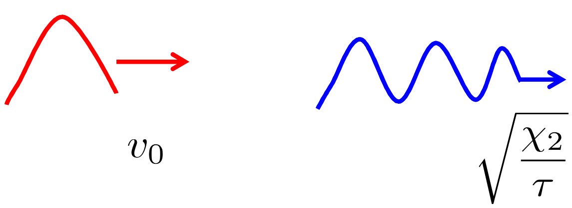
- Dynamics:

$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$

pulse

'Heat flux wave': $\sqrt{\frac{\chi_2}{\tau}}$
telegraph → wavy feature

two characteristic propagation speeds



→ In short response time (usual) heat flux wave propagates faster

→ In long response time, heat flux wave becomes slower and pulse starts overtaking.
What happens???

Analysis of heat avalanche jam dynamics

- In large tau limit, what happens? → Heat flux jams!!

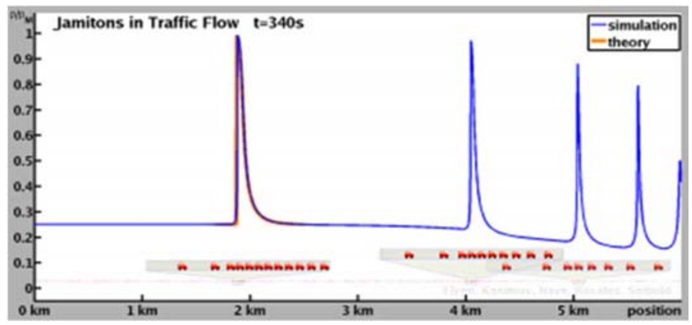
- Recall plasma response time akin to driver's response time in traffic dynamics

- negative heat conduction instability occurs (as in clustering instability in traffic jam dynamics)

$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$
$$\rightarrow \underline{(\chi_2 - v_0^2 \tau)} \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T}$$

<0 when overtaking

→ clustering instability



n.b. akin to negative viscosity instability of ZF in DW turbulence

instead ZF as secondary mode in the gas of primary DW

➔ Heat flux 'jamiton' as secondary mode in the gas of primary avalanches

Analysis of heat avalanche jam dynamics

- Growth rate of the jamiton instability

$$\gamma = -\frac{1}{2\tau} + \frac{1}{2\tau}\sqrt{\frac{r+1}{2} - 2\tau\chi_2 k^2\left(1+\frac{\chi_4 k^2}{\chi_2}\right)} \qquad r = \sqrt{\left\{4\tau\chi_2 k^2\left(1+\frac{\chi_4 k^2}{\chi_2}\right)-1\right\}^2 + 16 v_0^2 k^2 \tau^2}$$

- Threshold for instability

$$\tau > \frac{\chi_2}{v_0^2}\left(1+\frac{\chi_4 k^2}{\chi_2}\right)$$

→ critical minimal delay time

n.b. $1/\tau = 1/\tau[\mathcal{E}]$

→ clustering instability strongest near criticality

- Scale for maximum growth

$$k^2 \cong \frac{\chi_2}{\chi_4}\sqrt{\frac{\chi_4 v_0^2}{4\chi_2^3}}$$

from $\frac{\partial\gamma}{\partial k^2} = 0 \;\Rightarrow\; 8\tau\frac{\chi_4^2}{\chi_2}k^6 + 4\tau\chi_4 k^4 + 2\frac{\chi_4}{\chi_2}k^2 + 1 - \frac{v_0^2\tau}{\chi_2} = 0$

→ staircase size, $\Delta^2_{stair}(\delta T)$, δT from saturation: consider shearing

Scaling of characteristic jam scale

- Saturation: Shearing strength to suppress clustering instability

Jam growth → profile corrugation → ExB staircase → $v'_{E\times B}$

→ estimate, only

→ saturated amplitude: $\frac{\delta T}{T_i} \sim \frac{1}{v_{thi}\rho_i}\sqrt{\frac{\chi_4}{\tau}}$

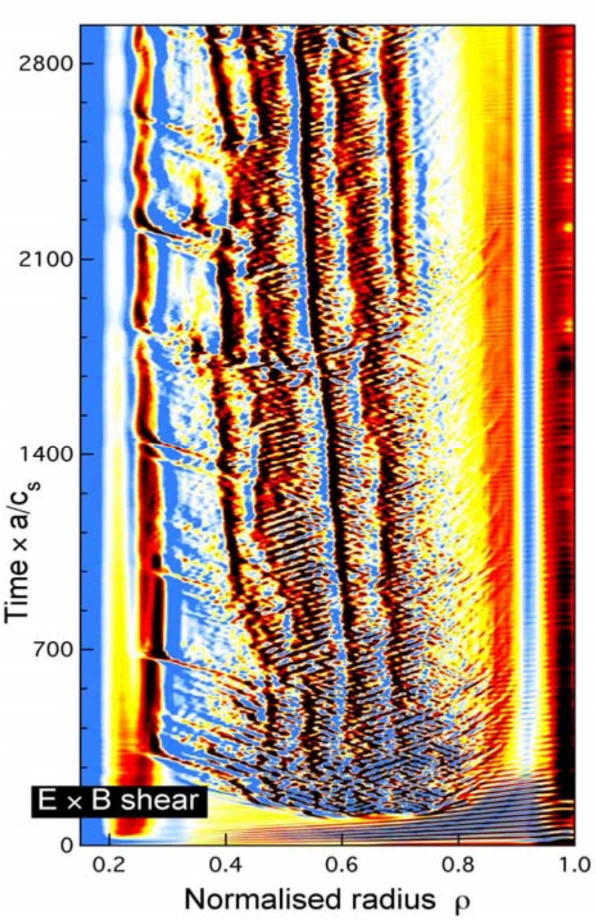
- Characteristic scale

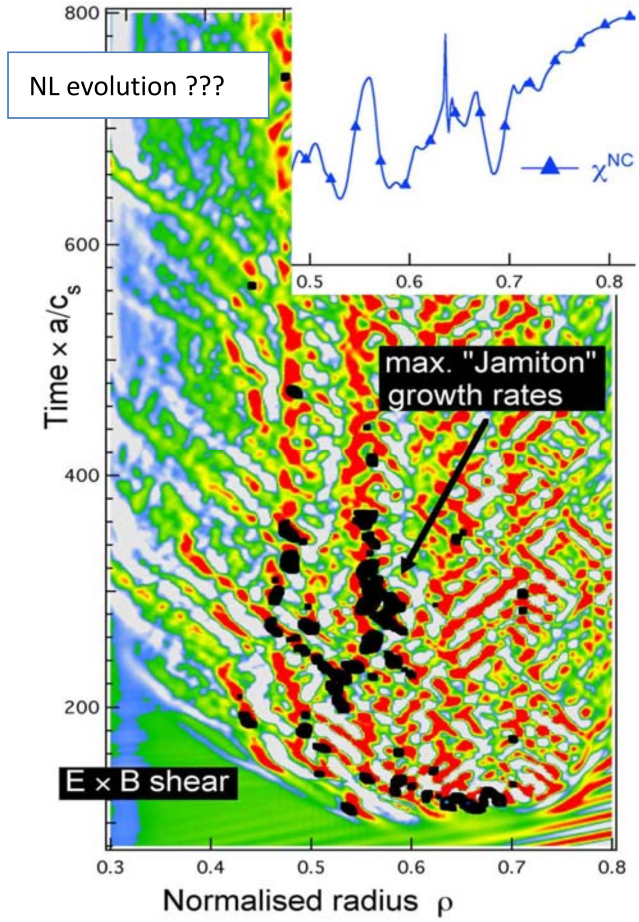
$$\Delta^2 \sim k^{-2}(\delta T) \sim \frac{2v_{thi}}{\lambda T_i}\rho_i\sqrt{\chi_2 \tau} \qquad \chi_2 \sim \chi_{neo}$$

- Geometric mean of ρ_i and $\sqrt{\chi_2 \tau}$: ambient diffusion length in 1 relaxation time

- ‘standard’ parameters: $\Delta \sim 10\Delta_c$

Jam growth qualitatively consistent with staircase formation





outer radius:
large chi
→ smear out instability
or
→ heat flux waves propagate faster
→ harder to overtake, jam

good agreement in early stage

Dif-Pradalier ’13 caveat: based on model with compressional waves

$\mathbf{E} \times \mathbf{B}$ staircase visible on fluctuation correlations

Direct exp. characterisation difficult:

flows, profiles & gradients

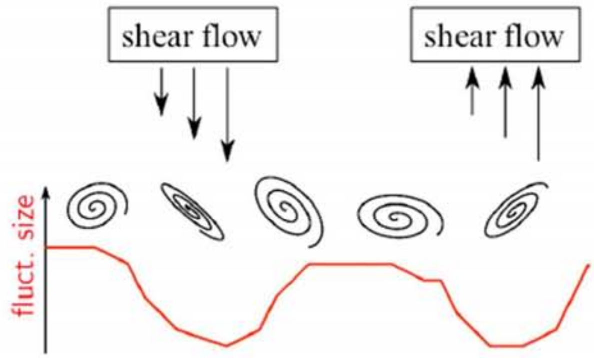
Shear layers in staircase:

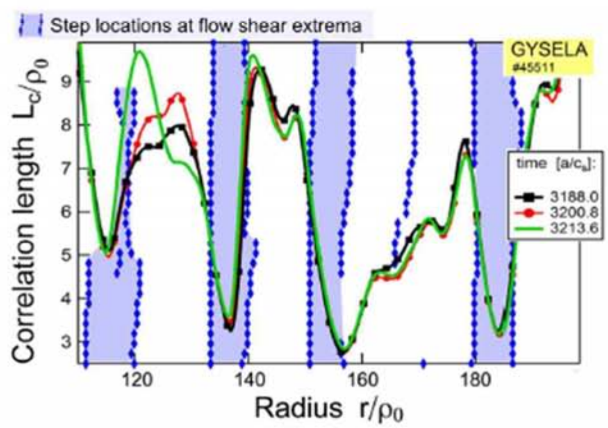
- eddies stretched, tilted, fragmented
- predict quasi-periodic decorrelation turbulent fluct.

$$\mathcal{C}_\phi(r,\theta,t,\delta r) = \frac{\langle \tilde{\phi}(r,\theta,t)\,\tilde{\phi}(r+\delta r,\theta,t)\rangle_\tau}{\left[\langle \tilde{\phi}(r,\theta,t)^2\rangle_\tau\,\langle \tilde{\phi}(r+\delta r,\theta,t)^2\rangle_\tau\right]^{1/2}}$$

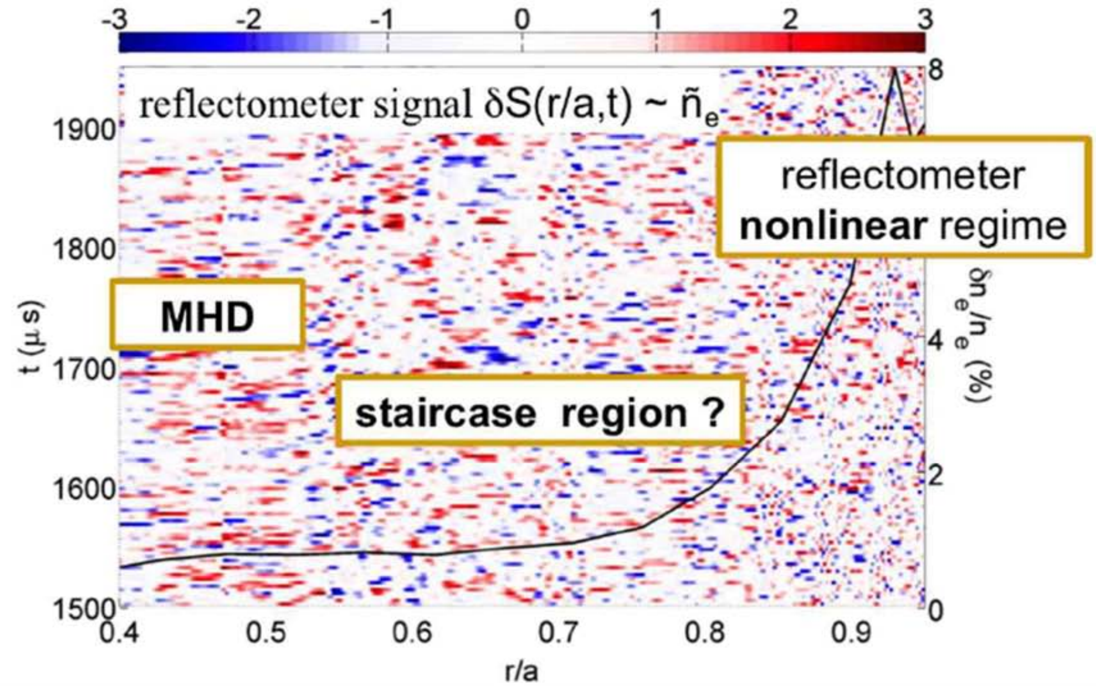
➥ $\mathcal{C}_\phi = 1/2$ when $\delta r = L_c$

➥ testable with fast-sweeping reflectometry





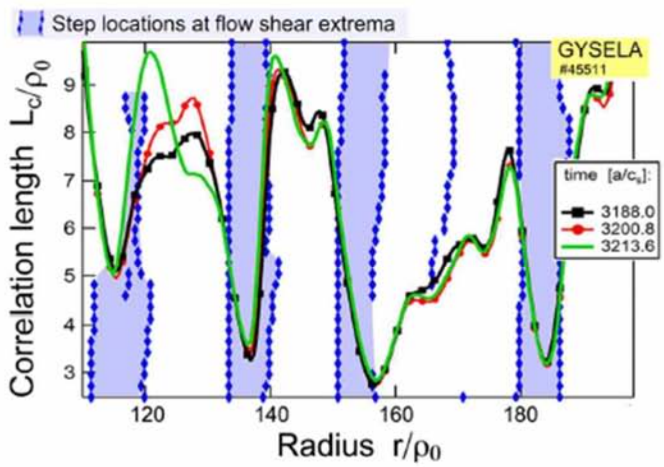
Moderate fluctuation level & MHD-free plasmas: optimal for staircase observation

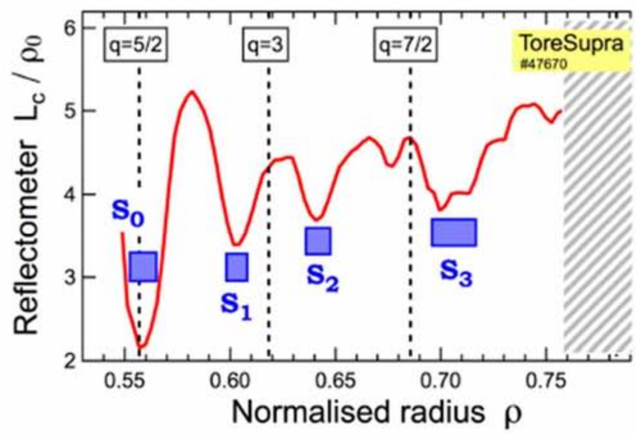


fast-sweeping reflectometry on **Tore Supra** [Clairet RSI 10, Hornung PPCF 13]

- ➡ localised measure, fast ($\sim \mu$s), sweeping in X–mode : full radial profile δn
- ➡ **routinely estimate** L_c

Staircase predicted. . .then observed experimentally





- Large set: 179 staircase steps, so far [Dif-Pradalier PRL 15 & Hornung, in prep.]
  - quasi-regularly spaced radial local minima of L_c
  - reproducible: not random & robust w.r.t. definition of L_c
  - tilt consistent with flow shear around minima
  - no correlation to local q rationals ➠ rules MHD out
  - consistent width [$\sim 10\rho_i$] & spacing [meso.] of local L_c minima

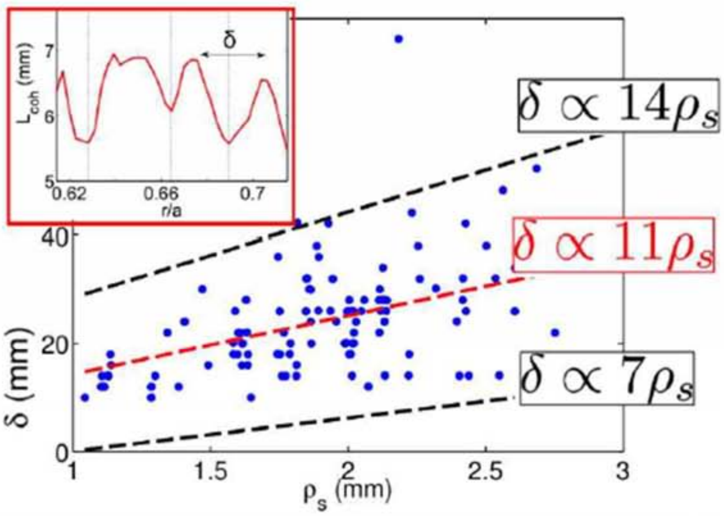
When **theoretical predictions** lead to **experimental discovery**

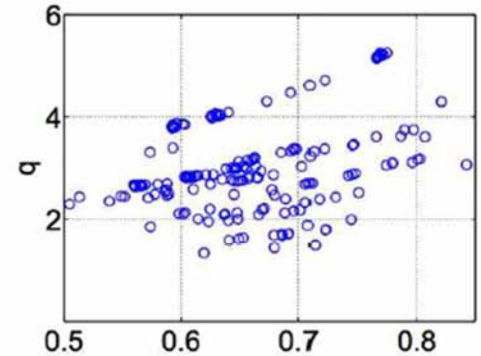
- flow width $\delta \sim 11\rho_i$ consistent with GYSELA obs. & ZF measurements

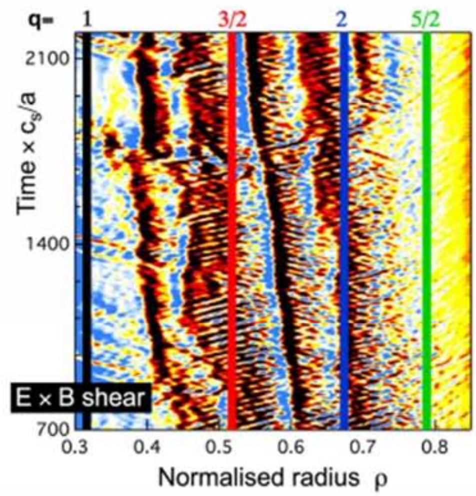
  [Fujisawa PRL 04]

- turbulence-borne ➠ not MHD

  [Dif-Pradalier PRL 15 & Hornung, in prep.]







Summary

- A model for ExB staircase formation

  - Heat avalanche jam → profile corrugation → ExB staircase

  - model developed based on analogy to traffic dynamics → telegraph eqn.

- Analysis of heat flux jam dynamics

  - Negative conduction instability as onset of jam formation

  - Growth rate, threshold, scale for maximal growth

  - Qualitative estimate: scale for maximal growth $\Delta \sim 10\Delta_c$

    → comparable to staircase step size

Ongoing Work

- This analysis ↔ set in context of heat transport
- Implications for momentum transport? ➔
  - consider system of flow, wave population, wave momentum flux
  - time delay set by decay of wave population correlation due ray stochastization → elasticity
  - flux limited PV transport allows closure of system

Results:

- Propagating (radially) zonal shear waves predicted, as well as vortex mode
- For τ_{delay} larger, Z.F. state transitions to LCO, rather than fixed point
- τ_{delay} due elastization necessarily impacts dynamics of L→I→H transition

Some Relevant Publications

- Y. Kosuga, P.H. Diamond, O.D. Gurcan; Phys. Rev. Lett. 110, 105002 (2013)
- O.D. Gurcan, P.H. Diamond, et al, Phys. Plasmas 20, 022307 (2013)
- Y. Kosuga, P.H. Diamond, Phys. Plasmas (2014)
- Y. Kosuga: Invited Talk, 2013 APS-DPP Meeting
- Z.B. Guo, P.H. Diamond, et al; Phys. Rev. E, in press (2014)
- Z.B. Guo, P.H. Diamond; Phys. Plasmas (2014)
- G. Dif-Pradalier, Phys. Rev. E (2010, Phys. Rev. Lett. (2015)
- P.-C. Hsu, P.H. Diamond, PoP, Phys Rev. E (2015)